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M. A./M.Sc. (THIRD SEMESTER)
EXAMINATION, Dec. - Jan., 2021-22
(MATHEMATICS)
PAPER THIRD (C)
(FUZZY SET AND THEIR APPLICATION)

[Time : Three Hours] ***[Maximum Marks : 80]***

Note : Attempt all sections as directed.

Section - A
(Objective/Multiple Choice Questions)

(1 mark each)

Note: Attempt all questions.

1. If A and B are two fuzzy sets in *X*, then for all $x \in X$, their standard fuzzy intersection is defined as $(A \cap B)(x)$ = ...
2. If A and B are any two fuzzy sets in *X*, then $A \cup (A \cap B) = A$ is known as the law of ....
3. Every fuzzy complement has at most .... equilibrium.

4. The property of *t*-conorm $u(a,a) > a$ for all $a \in [0,1]$ is known as.......... .
5. If $f : X \to Y$ is a crisp function, $A \in f(x)$ and $\alpha \in [0,1]$, then ${}^{\alpha}[f(A)] = f({}^{\alpha}A)$. (True/False)
6. A normal fuzzy set *A* on *R* such that ${}^{\propto}A$ is a closed interval for each $\alpha \in (0,1]$ and ${}^{0+}A$ bounded is called a ....
7. If ${}^{\alpha}A = [2\alpha - 1, 3 - 2\alpha]$ and ${}^{\alpha}B = [2\alpha + 1, 5 - 2\alpha]$, then ${}^{\alpha}(A - B) =$ ....
8. If A and B are fuzzy numbers, then for each $Z \in R$, A + B is a fuzzy set on *R* defined by (*A* + *B*) (z) = ...
9. If R(*x*, *y*) is a fuzzy relation, then for each $x \in X$, dom R (*x*) = ...
10. The standard composition of two binary fuzzy relations P(x, y) and Q(y, z) produces a binary fuzzy relation R(x, z) defined by ....
11. If R(x, x) is a fuzzy relation such that $R(x,x) \neq 1$ for some $x \in X$, then *R* is called .... relation.
12. A fuzzy relation *R*(*x*, *x*) is called antitransitive if ....

13. A binary relation *R*(*x*, *x*) that is reflexive and symmetric is called a tollerance relation. (True/False)

14. If $R = \begin{array}{c} \\ a \\ b \\ c \end{array} \begin{array}{c} \begin{array}{ccc} a & b & c \end{array} \\ \begin{bmatrix} 1 & .8 & 0 \\ .8 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \end{array}$ is a binary fuzzy relation on *X* = {*a, b, c*}, then $^{\wedge}R$ =............... .

15. X = B - A is the solution of the fuzzy equation A + X = B. (True/False)

16. If R(x, x) and Q(y, y) are fuzzy binary relations, then a function $h: x \rightarrow y$ is said to be a fuzzy homomorphism if.....................

17. For a given belief measure Bel, the corresponding basic probability assignment m is determined for all $A \in P(x)$ by the formula m(A) = ...

18. For all $A \in P(x), Pl(A) = 1 - Bel(\bar{A})$. (True/False)

19. Total ignorance is expressed in terms of the basic assignment by m (X) = 1 and m(A) = 0 for all $A \neq X$. (True/False).

20. For every $A, B \in P(x)$, Nec $(A \cup B)$ = min [Nec(A), Nec(B)]. (True/False)

Section - B

(Very Short Answer Type Questions)

(2 marks each)

Note- Attempt all questions.

1. If $A = .4/a + .3/b + .2/c + 0/d + 1/e$, then find $^{0.2+}A$.
2. Define cartesian product of two fuzzy sets.
3. Define fuzzy number.
4. Define difference of two fuzzy numbers.
5. Define asymmetric fuzzy relation.
6. Define upper bound for a fuzzy set.
7. Define fuzzy relation equations.
8. Define fuzzy measure.

Section - C

(Short Answer Type Questions)

(3 marks each)

Note- Attempt all questions.

1. Prove that the standard fuzzy union is the only idempotent *t*-conorm.

2. Prove that every fuzzy complement has at most one equilibrium.
3. If $f : x \rightarrow y$ is a crisp function and $B_1, B_2 \in f(y)$, then prove that $B_1 \subseteq B_2 \Rightarrow f^{-1}(B_1) \subseteq f^{-1}(B_2)$.
4. Using fuzzy numbers, define the concept of a fuzzy cardinality for fuzzy sets that are defined on finite universal sets.
5. Define preordering or quasi-ordering type of binary fuzzy relation R(x, x).
6. Draw the graph of the following fuzzy relation on X = {a, b, c, d, e}:

$$R = \begin{array}{c} \\ a \\ b \\ c \\ d \\ e \end{array} \begin{array}{c} \begin{array}{ccccc} a & b & c & d & e \end{array} \\ \begin{bmatrix} 1 & .7 & 0 & 1 & .7 \\ 0 & 1 & 0 & .9 & 0 \\ .5 & .7 & 1 & 1 & .8 \\ 0 & 0 & 0 & 1 & .2 \\ 0 & .1 & 0 & .9 & 1 \end{bmatrix} \end{array}$$

For $\propto$ = .7 and $\propto$ = .9.

7. If $P \overset{i}{o} Q = R$ and $S(Q, R) \neq \phi$, then prove that $\hat{P} = (Q_0^{\omega i} R^{-1})^{-1}$ is the greatest member of $S(Q,R)$.
8. Let a fuzzy set F be defined on N by $F = \frac{.4}{1} + \frac{.7}{2} + \frac{1}{3} + \frac{.8}{4} + \frac{.5}{5}$ and $A(x) = 0 \ \forall x \notin \{1,2,3,4,5\}$,. Determine Nec(A) and Pos (A) induced by F for all $A \in P(\{1,2,3,4,5\})$.

Section - D

(Long Answer Type Questions)

(5 Marks each)

Note: Attempt all questions.

1. Let i_w denote the class of yager t-norms, then prove that $i_{\min}(a,b) \leq i_w(a,b) \leq \min(a,b)$ for all $a,b \in [0,1]$.

Or

State and prove first decomposition theorem on fuzzy sets.

2. Solve the fuzzy equation A + X = B, where $A = \frac{.2}{[0,1]} + \frac{.6}{[1,2]} + \frac{.8}{[2,3]} + \frac{.9}{[3,4]} + \frac{1}{4} + \frac{.5}{[4,5]} + \frac{.1}{[5,6]}$ and

$$B = \frac{.1}{[0,1]} + \frac{.2}{[1,2]} + \frac{.6}{[2,3]} + \frac{.7}{[3,4]} + \frac{.8}{[4,5]} + \frac{.9}{[5,6]} + \frac{1}{6} + \frac{.5}{[6,7]} + \frac{.4}{[7,8]} + \frac{.2}{[8,9]} + \frac{.1}{[9,10]}$$

are two fuzzy numbers.

Or

Let R be a reflexive fuzzy relation on X^2, where $|X| = n \geq z$. Then prove that $R_{T(i)} = R^{n-1}$.

3. Explain projections and cylindrical extensions with suitable example.

Or

Show that for every fuzzy partial ordering on *X*, the sets of undominated and undominating elements of *X* are nonempty.

4. Solve the fuzzy relation equation.

$$p \circ \begin{bmatrix} .9 & .6 & 1 \\ .8 & .8 & .5 \\ .6 & .4 & .6 \end{bmatrix} = [.6 \ \ .6 \ \ .5]$$

For the max-min composition.

Or

Let $X = \{a, b, c, d\}$. Given the basic assignment $m(\{a, b, c\}) = .5$, $m(\{a, b, d\}) = .2$, and $m(X) = .3$, determine the corresponding belief and plausibility measures.